Toward more consistent pipe stress analysis

Presented are some guidelines in applying stress intensification factors to piping weight loading and small branch connections. It is hoped this information will alleviate the controversy and lead to standardization

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IN PIPING DESIGN HISTORY, 1955 is a monumental year. In that year the stress range concept was formally recognized by the Code for Pressure Piping[1] as the basis for evaluating thermal expansion stress. Although the code has been expanded and clarified over the years, there are still unsettled arguments regarding application of the code in certain areas. Two areas where inconsistencies still exist are stress intensification factors for weight and other steady loadings and stress intensification factors for small branch connections. These areas will explored along with suggestions for applying the code.

- **Stress intensification factor for weight and other steady loadings.** The stress intensification factors given in the code are intended for flexibility analyses. No specific intensification factor for weight and occasional loadings is mentioned in the Chemical Plant and Petroleum Refinery Piping Code.[2] Due to this tacit position of the code, piping designers are divided in actual practice. Some designers will apply the code stress intensification factors to all categories of loads, while many other designers tend to ignore the stress intensification factors completely in steady load analysis. One component acceptable to one designer can be rejected by others due to different opinions in the interpretation.

- **Small branch connections.** The stress intensification factors given in the code for branch connections are derived from full size branch connections. These factors, although applicable to small branch connections, can become excessively conservative for small connections on big pipes. Because of the apparent overconservatism, designers often ignore stress intensification factors at small branch connections. However, practices are never consistent. For instance, it is easy to see that the stress intensification due to a ¾-inch connection can be ignored in the analysis of a 20-inch header, but for a 3-inch connection, the factors to apply will differ among designers.

Stress intensification factors given in the code for branch connections can be too conservative for small connections on large pipes

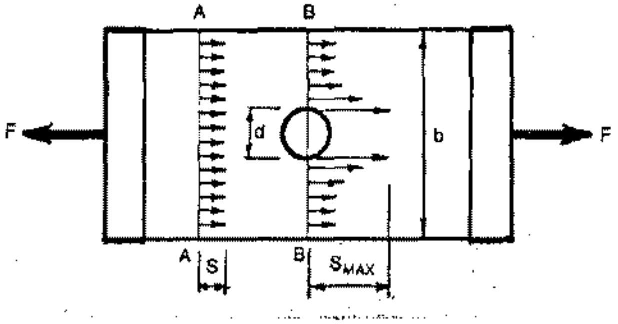

Fig. 1—Stretching a bar with a small hole

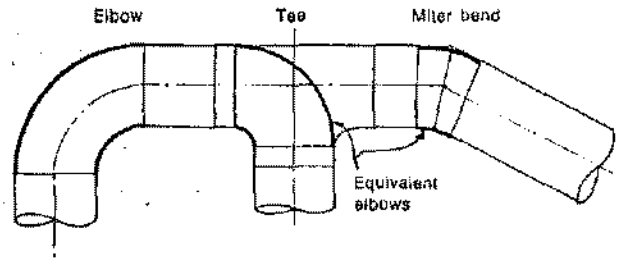

Fig. 2—Equivalent elbows

These two examples are related to the application of stress intensification factors. Apply or not to apply very often means several times difference in the allowable loads. These are determined solely by designers' personal preferences and inclinations. A more consistent approach needs to be developed and adopted.

STRESS INTENSIFICATION

When a structural member is stretched, stress in the main uniform section can normally be calculated by simple formulas, but the stress in a locally notched or stiffened discontinuous section is either very complicated or impossible to calculate. For practical design purpose, stress at the discontinuous section is estimated by applying a stress intensification factor over the stress calculated at the main uniform section. This stress intensification can be derived theoretically or determined by test.

At a structural discontinuity, stress intensification can be quite different for different types of loading. Fig. 1 shows a long rectangular bar with a small hole in the middle of the section. At Section A-A outside the influence of end fixtures and the hole, the stress is uniformly distributed at a magnitude of $S = F/(bt)$. But at Section B-B, due to discontinuity in strain flow, the stress is unevenly distributed. A maximum stress, S_{max}, of about three times the uniform stress occurs at the edges of the hole. The stress decreases very rapidly at points away from the edge of the hole. Theoretically, the hole has created a stress intensification factor of three, but its significance is different for different materials.

For a brittle material such as glass, the hole will degrade the bar to one-third its original strength because it fails as soon as the maximum stress reaches failure stress. Piping materials, on the other hand, are normally very ductile, and a considerable amount of yielding takes place before the member fails. With ductile materials the stress intensification needs to be interpreted in two different categories, namely steady and cyclic.

Steady loading. Under steady loading the highly localized stress concentration will be redistributed to the adjacent area once the local stress reaches the yield point. Eventually the load will spread evenly to the whole cross-section before the bar fails. The important stress is the redistributed stress prior to the failure. Since the redistributed stress is essentially the average stress, the stress intensification factor for steady loading is

$$i_s = \frac{F/(b-d)}{F/(b)} = \frac{b}{(b-d)} \qquad (1)$$

which is entirely due to reduction of the cross-sectional area.

Cyclic loading. Under cyclic loading the member fails due to fatigue. Since the primary measure of fatigue failure is the local strain range per cycle, redistribution of stress due to plastic flow is not very important. Therefore the stress intensification factor for cyclic loading is

$$i_c = S_{max}/S \qquad (2)$$

which is the measure of the maximum local strain. S_{max} is the maximum equivalent elastic stress rather than the actual stress.

Elbow stress intensification factor. In piping stress analysis, the elbow stress intensification factor is particularly important not only because the elbow constitutes a major portion of the system but also because it is the basis for deriving the stress intensification factor for other component shapes. For instance, Markl[3] successfully used elbow analogy to correlate his fatigue test results on tees and miter bends. Using the equivalent elbows as shown in Fig. 2 and making adjustments for actual crotch radius and thickness, a set of stress intensification factors was constructed using a single flexibility characteristic parameter, h. A detailed discussion on elbow characteristics is beneficial in understanding the general trend of all components.

An elbow behaves very differently from a straight pipe in resisting bending moments. When a straight pipe is bent, its cross-section remains circular and stress increases linearly with distance from the neutral axis. However, when an elbow is bent as shown in Fig. 3, the cross-section deforms to an oval shape. This ovalization is due to less rigidity at extreme fibers in the tangential t-t direction, and less energy being needed for the elbow to assume an oval shape than to maintain a circular cross-section. Top and bottom portions of the pipe wall simply buckle in to escape from carrying their proper share

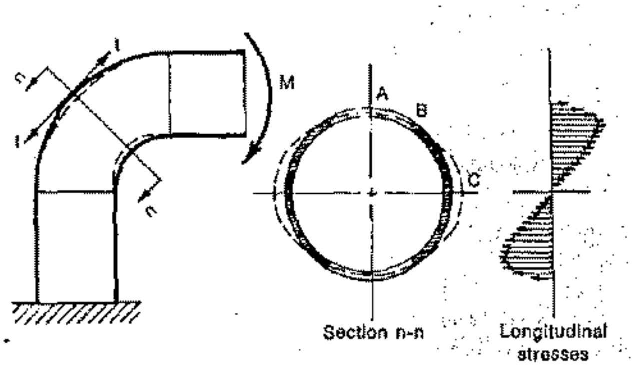

Fig. 3—Stress deformation of an elbow

of the load. The bending moment is resisted essentially by the shaded effective section. The maximum stress point is shifted from Point A to the effective extreme Point B. As the cross-section ovals, a local bending stress is also produced around the circumference. The maximum circumferential stress occurs at Point C where the radius of curvature is the smallest.

Mathematically the maximum longitudinal stress and circumferential stress can be calculated by using the following stress intensification factors:[4]

$$\left.\begin{array}{l}\beta_i = 0.84/h^{2/3}\\ \gamma_i = 1.80/h^{2/3}\end{array}\right\}\quad \text{in-plane bending} \qquad (3)$$

$$\left.\begin{array}{l}\beta_o = 1.08/h^{2/3}\\ \gamma_o = 1.50/h^{2/3}\end{array}\right\}\quad \text{out-plane bending} \qquad (4)$$

The experimentally measured distributions of the longitudinal and circumferential stresses of a 30-inch pipe elbow subject to in-plane bending[5] are shown in Fig. 4. Maximum circumferential stress is normally greater than the maximum longitudinal stress. However, the nature of the two stresses is quite different. The longitudinal stress is a membrane stress working directly against the moment, while the circumferential stress is a skin bending stress resulting from local deformation.

Code stress intensification factors. The stress intensification factors given in the code[2] are intended for thermal expansion and other displacement loads. The nature of thermal expansion load is different from that of weight and other sustained loads. Thermal expansion is self-limiting. It is a strain controlled loading such that once the strain reaches a point large enough to compensate for the expansion, growth stops regardless of the actual stress developed in the system. It can not normally cause any structural damage in one single application, but can cause fatigue failure through repeated expansion and contraction cycles. Therefore, for evaluating thermal expansion, the stress intensification factor is determined by the ratio of the stress causing failure over a given number of cycles in a straight pipe to the stress causing failure at a component subject to an equal number of stress cycles. Code stress intensification factors are cyclic or fatigue stress intensification factors in which the local peak stress is governing.

Theoretically these intensifications are equal to the maximum stress intensification existing in any region and direction within a component. In an elbow, for instance, the circumferential stress intensification factors $1.80/h^{2/3}$ and $1.50/h^{2/3}$ for in-plane and out-of-plane bendings, respectively, should be used. However, intensive fatigue tests on various components[3] have shown that by using unity as the fatigue life of girth welded or clamped pipe, the effective stress intensification factors of elbows in bending fatigue were about half the theoretical value. By dividing the theoretical factor by two, the code stress intensification factor for elbows is as follows:

In-plane stress intensification factor

$$i_i = 0.90/h^{2/3} \qquad (5)$$

Out-of-plane stress intensification factor

$$i_o = 0.75/h^{2/3} \qquad (6)$$

The stress intensification factors for other components are derived by using elbow analogy correlated with test results.

STRESS INTENSIFICATION FACTORS FOR WEIGHT AND OCCASIONAL LOADS

No stress intensification factor is explicitly stated in the Chemical Plant and Petroleum Refinery Piping Code for weight and occasional loads. Weight and wind are sustained loadings. They are not self-limiting, and always require a static equilibrium between the stress developed in the component and the load applied. Once yield point or collapse load is reached, the component will fail regardless of the amount of deformation that has occurred. Therefore, the stress to be considered in weight and other sustained loadings should possess the following characteristics:

► The stress is in a direction directly against the loading. Only the stresses acting against the load are load-carrying stresses.

► The stress is the average stress across the wall thickness. The average stress is actually the remaining stress available for external equilibrium after the internal mutual cancellation.

Theory and experiment indicate the same code stress intensification factors intended for flexibility analysis should be used in weight, occasional and other sustained load analyses

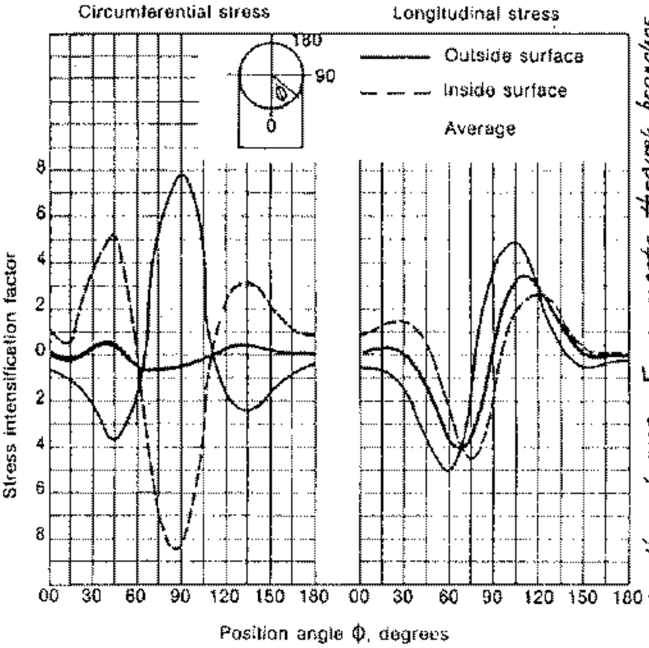

Fig. 4—Variation of stress around the circumference of an elbow with a 30-inch OD, 0.515-inch wall and a 45-inch bend radius.

From these two criteria and referring to Fig. 4, it can be concluded that in an elbow the stress intensification for weight and other sustained loads should be approximately equal to the longitudinal stress intensification. The higher intensification in the circumferential direction is not important here, because it is not in the loading direction and has very small average stress.

Since the stress-raising factor in a girth weld will not significantly affect the load-carrying capacity, the theoretical β stress intensification factors shown in Equations 3 and 5 can be used directly even in reference to girth welded pipe. By comparing Equations 3, 4, 5 and 6 it is clear that the code stress intensification factors can also be used for weight and other sustained loadings without losing much accuracy.

These are purely mathematical deductions which need to be substantiated by experiments. The stress intensification measure of a component subject to a sustained loading is its collapsing strength. Bolt and Greenstreet[6] have made substantial tests in determining the collapse loads for elbows. Schroeder,[7] on the other hand, has done the same for branch connections. Some of their test results are summarized in Table 1. The elbow collapse moments shown in the table are taken at the center of elbow arc rather than at the loading end of elbow edge as in Literature cited 6. The code stress intensification factors for the tested special tees are calculated by assuming a square reinforcing pad having a diagonal dimension the same as the throat dimension of the crotch radius.

From Table 1, again it is apparent that the stress intensification factors as represented by M_p/M_c are very close to the code stress intensification factors intended for cyclic loadings. The somewhat larger factor experienced by the stainless steel elbow appears to be caused by the inherent round-house stress-strain curve of the material which flattens the load-deflection curve at an earlier stage.

This discussion is mainly for moments through run. For moments through branches the actual SIF can be bigger for small branches. $i_b/i_o = 0.5$ is about the worst for through brand load.

STRESS INTENSIFICATION FACTORS FOR SMALL BRANCH CONNECTIONS

The code stress intensification factors for tees and branch connections were derived from full-sized branch connections. In applications where the branch size is much smaller than the run size, application of these factors can be grossly too conservative. Although Code Case No. 53, which was subsequently incorporated in the code, provided some relief to the branch itself, it did nothing to relieve the moment load transferred through the run pipe. Therefore, when it comes to the run moment, the current practice is to completely ignore the very small branches which are defined rather arbitrarily by individual designers.

Basically, the present code requires that a uniform stress intensification factor be used for moments acting both through the branch and through the run. For a reduced outlet the section modulus used in determining branch stress can use so-called effective branch wall thickness instead of the actual thickness. The effective branch thickness, T_s, is the lesser of run thickness, T, and the product of out-of-plane stress intensification and branch thickness i_oT_b. In other words, the stress intensification based on branch section modulus can be reduced by a factor of T_b/T for moments acting through branch. There is still no relaxation given to the moments carried straight through the run pipe.

Empirically, the stress intensification factor for an out-of-plane bending moment applied to the branch pipe can be expressed as[8]

$$i_b = A(r/T)^{2/3}(r_b/r)^{1/2}(T_b/T) \tag{9}$$

Except for the $(r_b/r)^{1/2}$ term, Equation 9 is an exact expression of code requirements. The term $A(r/T)^{2/3}$ is the code stress intensification factor with $A = 0.335$ for a welding tee and so forth, and (T_b/T) is the effective thickness factor stipulated in Code Case No. 53. Since the (T_b/T) factor has been included in the code definition of effective branch wall thickness, it can be removed from the equation. By rearranging the equation, we have

$$i_b = i_o(r_b/r)^{1/2} \tag{10}$$

where i_o is the code stress intensification factor. Without the effective thicknes factor, Equation 10 can also be used for moments acting through the straight runs. This equation can serve as a gradual transition from full-sized outlets to small connections.

CONCLUSIONS

Currently there is no explicit statement in the Chemical Plant and Petroleum Piping Code requiring the application of a stress intensification factor in weight and other sustained load analyses. Application of these factors is therefore determined by the design specification prepared

TABLE 1—Collapse moments on elbows and tees

Test piece	Material	Yield stress (ksi)	Moment direction	Collapse moments (in.-kip)		M_p/M	Code stress intensif. i
				Test piece M	Straight pipe M_p		
6-in. Sch. 40 LR elbow..............	ASTM A-106B	50.0	In-plane open	235	564	2.4	2.27
6-in. Sch. 40 LR elbow..............	ASTM A-106B	50.0	In-plane close	208	564	2.71	2.27
6-in. Sch. 40 LR elbow..............	ASTM A-106B	50.0	Out-plane	234	564	2.41	1.89
6-in. Sch. 80 LR elbow..............	ASTM A-106B	37.8	In-plane open	435	627	1.44	1.64
6-in. Sch. 80 LR elbow..............	ASTM A-106B	37.8	In-plane close	358	627	1.75	1.64
6-in. Sch. 80 LR elbow..............	ASTM A-106B	37.8	Out-plane	409	627	1.53	1.37
6-in. Sch. 40 SR elbow..............	ASTM A-106B	39.6	In-plane open	184	447	2.43	2.96
6-in. Sch. 40 SR elbow..............	ASTM A-106B	39.6	In-plane close	175	447	2.55	2.96
6-in. Sch. 40 SR elbow..............	ASTM A-106B	39.6	Out-plane	196	447	2.28	2.43
6-in. Sch. 40 LR elbow..............	ASTM A-312	37.7	In-plane close	117	426	3.64	2.27
3.49-in. OD 0.14-in. t tee............	AISI 1020C	25.0	In-plane	27	42.5	1.57	1.72
3.49-in. OD 0.14-in. t tee............	AISI 1020C	31.4	Out-plane	28	54	1.93	1.96

by the owner or its agent. However, there are widely divided opinions regarding the magnitude of the factors to be used. From the discussions presented in this article, it appears that both theory and experiment have indicated the same code stress intensification factors intended for flexibility analysis should also be used in weight, occasional and other sustained load analyses.

For branch connections, the code stress intensification factors were basically obtained from tests on full-sized outlet connections. In small-sized outlet connections, the code has provided some relief for moments acting through branches but no relief is given for moments acting through the straight runs. Although common practice is to ignore stress intensifications at very small branches, a guideline is needed for making the decision. With the unresolved situation that exists, a designer's rather arbitrary decision can artifically make a component several times weaker or stronger. This inconsistency can be greatly mitigated by multiplying the code stress intensification factor with a gradual size reduction factor $(r_b/r)^{1/2}$. This factor has been adopted in the Power Piping Code[9] for certain branch connections.

About the author

Liang-Chuan Peng *is a consulting engineer in Houston. He is currently on assignment at Pullman Kellogg's Piping Mechanics Section where he is working on special piping support and stress analysis. His more than 18 years of experience includes pipe stress analysis, specification and piping computer program development for Brown & Root, Bechtel Inc., Foster Wheeler, Taiwan Power Co. and many others. Mr. Peng holds a B.S. equivalent in mechanical engineering from Taipei Institute of Technology and an M.S. in mechanical engineering from Kansas State University. He is a registered engineer in Texas and California and a member of ASME. Mr. Peng is also the author of several technical papers and computer programs.*

NOMENCLATURE

$\beta = S_{l\,max}/S$, longitudinal stress intensification factor
$\gamma = S_{c\,max}/S$, circumferential stress intensification factor
$S_{l\,max}$ = Maximum longitudinal stress, psi
$S_{c\,max}$ = Maximum circumferential stress, psi
$S = M/Z$, equivalent bending stress developed in a straight pipe of identical cross-section, psi
M = Bending moment, in.-lb.
Z = Section modulus of the pipe section, in.^{3}
$h = TR/r^2$, the flexibility characteristic
R = Bend radius, in.
T = Wall thickness of the pipe, in.
r = Mean radius of the pipe cross-section, in.
i_b = Stress intensification factor for branch connection
r_b = Mean radius of branch pipe, in.
T_b = Thickness of branch pipe, in.
A = Empirical correlation constant.

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