ABSTRACT

This paper describes the development of computer methods and analytical procedures necessary to satisfy requirements for analysis of Class I nuclear piping systems in accordance with the ASME Section III Code (Code). Topics covered include formulation of a simplified method for thermal transient analysis; selection from available sources techniques for piping flexibility analysis; and interpretation of Code requirements in developing procedures for Class I stress analyses. Specific discussion is given to the development of a criterion for the spacing of lumped masses for dynamic analysis. A method for calculating moments at piping tees for those load cases without proper signs, and a criterion for selection of load case combinations to avoid over-conservatism in resulting pipe stresses are also presented. And finally, discussion is given to computer program verification and control needed to ensure quality control standards and conformance to the Code.

INTRODUCTION

Three basic conditions must be satisfied in order to qualify Class I nuclear piping systems to the requirements of the ASME Section III Code. Primary stresses, which include internal pressure, deadweight, earthquake inertia forces and applied forces, are limited to the "design yield stress" of the material. Design yield stress ranges from the actual yield stress for carbon and low alloy steels to about 135% yield stress for austenitic steels and some non-ferrous materials. Primary plus secondary stresses, which additionally includes thermal expansion and thermal transient loads, are allowed to reach 200% of the material design yield stress due to their self-limiting nature. Peak stress, which include effects of local stress concentrations and discontinuities, are limited by empirically derived Code fatigue curves.

An exact determination of these stresses, in terms of today's calculational techniques, would require detailed finite element modeling of localized regions of each component in the piping system, including coupled transient heat transfer and mechanical/thermal loadings. However, the size, number and complexity of piping systems in a typical nuclear plant makes such detailed modeling of all components impractical. Instead, the overall piping stresses are calculated at discrete points (of maximum stress) using comparatively simple beam type finite element models. Local stress concentrations due to non-uniformity in the piping cross-sections are accounted for through use of
stress intensity factors. Stresses in the pipe wall due to fluid temperature transients are calculated in separate heat transfer analyses. And finally, the continuum of the stress state in the piping with respect to time is approximated by analysis characterizing load conditions at critical time points.

Even using these methods to simplify the analysis, careful attention must be paid to the organization and efficiency of the procedures. The authors have participated in development of two computer codes now being used for performing this analysis. The TNEAT program calculates linear and nonlinear wall temperature gradients and discontinuity temperature differences in a pipe subject to a temperature transient in the contained fluid. The results from TNEAT form a portion of the input to NUPipe, a program which performs the piping flexibility analysis and evaluates resultant stresses in accordance with the specific requirements of Section III of the ASME Code. A description of the assumptions and methodology employed in each of these programs and the procedures for their use in the analyses of nuclear piping is given in the following sections.

**THERMAL TRANSIENT ANALYSIS**

**Thermal Transient Analysis**

**OMITTED**

**PIPING RESPONSE ANALYSIS**

**Introduction**

To demonstrate conformance of Class 1 piping systems to the requirement of the Code, piping system response to the following loads must be determined:

(a) Thermal expansion of the piping
(b) Deadweight of the piping and its content
(c) Relative anchor and support movements
(d) Externally applied static forces and moments
(e) Externally applied dynamic forces and moments
(f) Earthquake motions

Response to loadings (a), (b), (c) and (d) are determined from static analyses. Those of (e) and (f) are determined using dynamic analyses.

**Static Analyses**

In the development of structural mechanics, analyses of piping systems may have been the first subject for which the synthesis method was used. The systematic analysis of a piping structure using elbows and straight pipe sections
as building blocks was developed long before the birth of the commercial electronic digital computer. Following the commercial introduction of the digital computer in the 1950's, many general matrix methods have been developed. Brock and Chen (6) and (7) present two typical methods for use in the piping area. The solution methods have now been fully developed and can be grouped into two categories; the flexibility method and the stiffness method.

The Flexibility Method

The flexibility method can be expressed in matrix form as:

\[
[C] \{F\} = \{D\}
\]  
(7)

in which \([C]\) is the flexibility matrix, and \([F]\) and \([D]\) are the force and displacement vectors, respectively. By using a normalized coordinate system, a piping system can be reduced so that the assembled flexibility matrix \([C]\) will have a dimension only equal to the number of degrees of freedom constrained by anchors and supports. The displacement vector \([D]\) is initially calculated at all the constrained points assuming the constraints are removed. The constraint reactions are then calculated by solving equation (7) for the forces and moments required to bring the unconstrained displacements back to their constrained positions. The effect of externally applied loadings and dead-weight of the piping is treated by adding an equivalent displacement term in vector \([D]\). These procedures are described more fully in Reference (8). Since the size of the flexibility matrix \([C]\) is relatively small, this method is especially suitable for small computers. Most of the pipe stress computer programs developed in early 1960's (2), (10) and (11) used the flexibility method. Unfortunately, there are many topological and bookkeeping problems inherent in the application of the flexibility method. For example, the logic used in handling multi-connected loops is very complicated.

The Stiffness Method

The stiffness method as expressed in equation (8) is generally referred to as the direct stiffness method (not to be confused with the method of Chen (7)).

\[
[K] \{D\} = \{F\}
\]  
(8)

In equation (8), \([K]\) is the stiffness matrix of the piping system; and \([D]\) and \([F]\) are the displacement and force vectors, respectively. This method has long been developed for solutions to general structural mechanics problems, and its formulation can be found in any textbook on matrix methods of structural analysis (12) and (13). As opposed to the flexibility method, the stiffness method calculates the force vector \([F]\) for each load case and solves equation (8) for the unknown displacement vector \([D]\). Thermal effects, deadweight and support displacement loads are converted to an equivalent force vector in \([F]\). Internal pipe forces and stresses are then calculated by applying the displacement vector \([D]\) to the individual element stiffness matrices. This method has a simple straightforward logic for even complex systems. Multi-nested loops are handled just like ordinary branched systems. One disadvantage of this method is the size of stiffness matrix; generally equal to six times the number of nodes in the system. This is almost an order of magnitude higher than the size of the \([C]\) matrix used in the flexibility method. With this many simultaneous equations to solve, even if the band width of the stiffness matrix is optimized and only the banded elements are stored, the computer core storage required is considerably higher than that required in the flexibility method. The final stiffness size can be reduced somewhat, however, by use of the transfer matrix method. Recent advances in computer central core and auxiliary memory capabilities have lessened this disadvantage such that most recent developments in computational structural mechanics are based on the stiffness method. As a result, most of the pipe stress programs developed recently, (14), (15) and (16), use this method.
For the seismic analyses, the equilibrium equation is written as:

\[ [M] \ddot{\mathbf{D}} + [C] \dot{\mathbf{D}} + [K] \mathbf{D} = -[M] \ddot{\mathbf{D}} \]

in which \([M]\) is the mass matrix
\([C]\) is the viscous damping matrix
\([K]\) is the stiffness matrix
\(\mathbf{D}\) is the displacement vector
\(\dot{\mathbf{D}}\) is the velocity vector
\(\ddot{\mathbf{D}}\) is the ground acceleration vector

This equation is used to calculate the reaction forces and inertia forces for the flexibility method, or to calculate the modal displacements for the stiffness method. Internal pipe forces and stresses are then calculated using the static equations (7) or (8).

Equation (9) can be solved using one of three methods; namely direct integration, modal integration, or the response spectra method (17). The difficulties and additional costs associated with specifying a ground motion acceleration time history and subsequently determining the system response using either the direct integration or modal integration method has lead to the response spectra method becoming more or less an industry standard.

The response spectra method uses the modal superposition assumption that the response at each point in a system, at a given time, equals the sum of the response of each normal mode oscillation at that point at that time. The response spectra method removes the time dependent factor and only the maximum response of each normal mode is calculated. Techniques for modeling a piping system for dynamic analyses and determination of the resulting response using the method of modal combinations are discussed in the following subsections.

System Modeling Using the Lumped Mass Method

A uniform pipe theoretically has an infinite number of degrees of freedom which allow it to move in an infinite number of ways. Since certain patterns of motion do not cause any macroscopic structural effect, the pipe can be idealized into a finite number of degrees of freedom by lumping its mass into certain concentrated mass points. In order to minimize computer run time it is advantageous to lump the system into as few mass points as possible. Care must be taken, however, to ensure that the location of and spacing between mass points does not exclude some of the significant patterns of pipe motion.

In seismic analyses, it has been recognized that the normal mode oscillations with very high frequency contribute little to the overall system response. The highest frequency of oscillation that must be included is termed the cut-off frequency, which is about 25 cps to 35 cps depending on the type of the supporting structures. Once the cut-off frequency is defined, the mass must be lumped in such a way that all the normal mode oscillations with frequencies lower than the cut-off frequency be determined within an acceptable tolerance. This can be accomplished by limiting the distance between mass points. To establish this limiting distance, it is necessary to first visualize the half wave idealization of a natural mode shape.
As shown in Figure 6, by using Rayleigh's method (18), the natural frequencies of the two idealizations are calculated and compared with the exact frequency. It is clear that if two or more lumped masses are located in a half wave, the simulated frequency differs from the exact frequency by only 0.7%, which is considered acceptable for this type of analysis. The following method can thus be used to check the maximum allowable distance between two lumped masses.

Assuming \( Q \) is the cut-off frequency in cycles per second, then the half wave length of this oscillation is

\[
L = \left( \frac{1.57}{\text{Q}} \right)^{1/3} \left( \frac{\text{E}\text{EI}}{\text{W}} \right)^{1/3}
\]  \( (10) \)

where
- \( g \) = acceleration of gravity
- \( E \) = modulus of elasticity
- \( I \) = bending moment of inertia
- \( W \) = pipe weight per unit length

Since two masses are required in each half wave length, the maximum allowable mass point spacing is

\[
\text{(Mass Point Spacing)} = 1/2 \left( \frac{1.57}{\text{Q}} \right)^{1/3} \left( \frac{\text{E}\text{EI}}{\text{W}} \right)^{1/3}
\]  \( (11) \)

In addition to the above general criterion, at least one mass has to be lumped in each pipe run or branch in order to correctly simulate the inertia force distribution.

Modal Combination

Using the response spectra method, the calculated response of each normal mode of oscillation is the maximum value encountered during the time of excitation. As the maximum responses of all the normal oscillating modes do not occur at the same time, a statistical combination method is used to obtain the combined response. A generally accepted method is to take the square root of the sum of the square (SRSS) of all the quantities involved. For example, for
each normal oscillating mode, system response results from earthquake motions in three orthogonal directions, namely vertical and the two horizontal directions. Using SRS method, the response in each mode due to each direction of earthquake loading is calculated independently with the response combined as follows:

\[
R_i = \sqrt{R_{ix}^2 + R_{iy}^2 + R_{iz}^2}
\]

(12)

where \( R_i \) is the combined response of the \( i \)th normal mode, and \( R_{ix}, R_{iy} \) and \( R_{iz} \) are the response of the \( i \)th normal mode due to earthquake motions in X, Y, and Z directions respectively. Pipe deflections corresponding to \( R_i \) are determined at this stage such that pipe forces and moments can be calculated properly. It should be noted that if the pipe forces and moments are not calculated at this stage, but rather calculated later using the combined deflections from all the normal modes, the calculated pipe internal forces and moments will not always be correct. This is because the signs of the deflections have been lost during the combination process and the deflected shape cannot be determined uniquely.

After the response in each mode is calculated, the combined system response is given by:

\[
R = \sqrt{\sum_i R_i^2}
\]

(13)

Where \( R \) is the combined system response due to \( n \) normal oscillating modes.

Recently there have been some discussions regarding the adequacy of equation (13). It has been suggested that responses of normal modes having essentially the same natural frequencies will reach their maximum value at about the same time and should be combined directly. Therefore, in some design specifications, equation (13) has been revised as follows:

\[
R = \sqrt{\sum_{m}^{M} \left( \sum_{l}^{L} |R_{il}| \right)^2}
\]

(14)

in which \( M \) is the number of the oscillating groups and \( L \) is the number of the normal modes with closely spaced natural frequencies in each oscillating group. Normal modes with frequency differences less than 10% are generally considered being closely spaced.

It has been found that equation (14) will not be adequate for some special cases. This happens when the piping system is over supported. As shown in the typical response spectra curve of Figure 7, if a system is designed very stiff it can be approximated as a rigid body during an earthquake. Total system response will then be equivalent to the response caused by a constant acceleration \( A \). However, even if many normal oscillating modes have been included in the response spectra analysis the constant acceleration effect cannot be reached. Therefore, it may be advisable to revise equation (14) to read as:

\[
R = \sqrt{R_g^2 + \sum_{m}^{M} \left( \sum_{l}^{L} |R_{il}| \right)^2}
\]

(15)

Where \( R_g \) is the response to rigid body motion. \( R_g \) can be calculated by applying the constant acceleration \( A \) at each mass point.
Dynamic Force Analysis

One of the more troublesome areas encountered in piping stress analysis is that associated with dynamic force analysis such as loadings resulting from safety or relief valve blowdown. For this event, steam or water traveling through a piping system will result in a force application to the pipe wherever the direction of fluid flow is altered (i.e. at elbows, tees, etc.). Typical shapes of these fluid forces are shown in Figure 8. The dynamic load factor\(^2\) for this type of loading will generally vary from zero to two depending on the rise time and duration of the excitation force. Therefore, it is very difficult to justify a static equivalent analysis unless a conservative dynamic load factor of two is used. For this reason dynamic time-history analyses using either the direct integration or the modal integration method is generally required.

Figure 8 Typical Safety or Relief Valve Blowdown Forces

\(^2\)In this context the dynamic load factor is defined as the ratio of the equivalent static force producing the peak pipe response to the maximum fluid excitation force.
One of the difficulties encountered in the analysis of safety or relief valve blowdown occurs when a restraint is positioned at a location where the force is to be applied (also see Figure 8). If the restraint is very stiff and directly opposes the applied force, the localized response will be of high frequency. Consequently additional care must be taken when developing the mathematical model of the system in specifying mass point locations; calculating the high frequency modes of pipe vibration; and specifying a sufficiently small integration time step to correctly integrate equations of motion for the high frequency modal vibrations. Artificially reducing the restraint spring constant can sometimes be used to minimize the above problems. However, when using this method, analysis results should be carefully examined to ensure their validity.

PIPING STRESS ANALYSIS

Introduction

In performing Class I piping stress analysis, ASME Section III allows the use of one of three methods: simplified elastic-plastic analysis, detailed analysis or experimental analysis.

The simplified elastic-plastic analysis is fully described in Section III, Subarticle NB-3600. In this method, a set of stress indices are developed so that pipe stress can be calculated easily and conservatively. The stress orientations are ignored and only the resultant moment is used in the stress calculation. The ASME sample analysis (1) demonstrates the use of this method.

For those pipe components which do not satisfy the simplified elastic-plastic analysis, detailed stress analyses can be employed. The procedure to be used for a detailed analysis is specified in Subarticle NB-3200. In this type of analysis all the stress orientations are considered with resulting stress values based on actual component dimensions. Outside of elbow components remote from welds whose stress indices are specified by Subarticle NB-3600, almost all the component stresses must be determined by detailed finite element analysis or by experimental analysis. Therefore, it has generally been found that it is more economical to replace or modify a piping component which does not meet the simplified elastic-plastic analysis than it is to perform the detailed analysis. Since the simplified elastic-plastic analysis is the only method which is practical for large volume production analyses, the following discussions will be limited to this method.

Load Cases

As discussed in the section on thermal transient analysis, it is necessary to define and calculate the stress and stress range that a piping component will be subjected to as a piping system responds to transition between various operating conditions (states). Section III requires that all combinations of operating variables, such as temperature and pressure, from one operating state to another operating state have to be considered in the analysis. The method generally used computes the response from the zero state (i.e., cold shutdown) to the various operating states. Each state is defined as one load case and the stress ranges are calculated by taking the differences in stresses between any two load cases forming a pair. Although other approaches may also be acceptable, this one appears to be the most straightforward and can be easily adopted to computer analyses.

To illustrate this method, let's assume a particular piping system experiences the transient history shown in Figure 9. To determine the stress range between states C and D, the stress associated with a transition from zero state to state C is defined as one load case, while the transition from zero state to state D is defined as a second load case. The stress range between C and D is then the difference between the maximum (at state C) and the minimum (at state D) stresses. Thus, for example, if a piping system will experience 20 different operating conditions, 20 load cases will be defined.
Figure 9  Typical Temperature Transient History

When utilizing this method one caution should be mentioned. If we have an operating condition at E which is associated with a very low level of stress, the pairing of load cases C or D with E would artificially create many transients of high stress range that do not exist. Thus the following guidelines in defining load cases should be employed:

(a) For each plant transient, two load cases must be defined. One load case will be associated with the maximum values of temperature/pressure encountered in the transient (the up condition), while the second load case will be associated with minimum values of pressure/temperature in the transient (the down condition). The number of operating cycles for each load case will be equal.

(b) Since most piping systems have very high number of cycles of fluctuation near the steady state condition, it should be recognized that it is important to correctly define the number of operating cycles for the zero case (plant shutdown). The number of operating cycles for the zero case should be made equal to the number of normal and abnormal plant shutdowns.

(c) If there are a large number of operating cycles which exist at a very low response level (i.e. area E of Figure 9) these operating cycles can be either neglected or upgraded by superimposing them with pipe response at the normal operating condition.

Seismic Stress Range

In the ASME sample analysis (1), positive values of piping response resulting from the seismic event were combined with the normal operating condition to form a load case. Likewise negative seismic response was combined with the normal operating condition to form a second load case. These two artificially created load cases represent the upper and lower envelope of the response when an earthquake takes place during the normal operating condition. The fatigue analysis then can proceed considering these two load cases the same as any other load cases.

If the earthquake occurs during other than normal operating condition, the response has to be combined with the worst load case pair. Half range of the seismic response is added to the maximum pair response range. If the resulting stress range is less than the seismic response range, the seismic response range is used. In the event that the number of earthquake cycles is greater than the number of cycles for the worst load case pair, the additional number of cycles is carried over to the second worst load case pair and the seismic event is again superimposed on this transient. This process is repeated until all the earthquake cycles are used up.
Figure 10  Moments at Piping Tees

Tee Moment Calculation

At branch connections, the combined effect due to both run moment and branch moment must be used in the Class 1 stress evaluation. The branch moment is well defined in the ASME Section III Code as shown in Figure 10. However, the definition of the run moment \( M_r \) is often misunderstood. From ASME Section III Subarticle NB-3600 and ANSI B31.7, 1971, the run moment can be considered as the moment that exists in the run pipe that is not produced to balance the branch moment. According to NB-3600 the components \( M_{xr} \), \( M_{yr} \), and \( M_{zr} \) of the run are determined as follows:

If \( M_{i1} \) and \( M_{i2} \) have the same algebraic sign then \( M_{ir} = 0 \)

If \( M_{i1} \) and \( M_{i2} \) have different algebraic signs, then \( M_{ir} \) is the smaller of \( M_{i1} \) or \( M_{i2} \); where \( i = X, Y, Z \)

The procedure outlined works perfectly for the static load cases. However, if the seismic analyses are performed using the response spectra method, the rule can only be used when the stresses are evaluated at the level of the individual normal mode. If the stresses are evaluated at the level of the final combined result, the rule does not apply as the signs of the moment components are lost after the combination. A procedure to overcome this difficulty is described as follows:

For each direction, the maximum run moment component that is not used to balance the branch moment is,

\[
(M_{ir})_A = |M_{i1}| + |M_{i2}| - |M_{i3}|
\]  

(16)

It is also apparent that the run moment component that is not used to balance the branch moment should be balanced by the moment at the other end of the run. Therefore, the maximum run moment component that is not used to balance the branch moment should be less than or equal to the moment at either end of the run. That is,

\[
(M_{ir})_B = \text{Minimum of } |M_{i1}| \text{ or } |M_{i2}|
\]  

(17)

Since equations (16) and (17) represent the maximum possible conditions, the actual run moment component should be the minimum of the two. Hence,
The above procedure has been utilized in the NUPITE II computer program (16) developed by the authors for all load cases, including static load cases, which have no clear sign relations.

Program Verification

Verification of the NUPITE II computer program was made by comparison with results obtained using other matrix structural analysis programs with bench mark cases provided by the ASME (19) and with hand calculated solutions.

CLOSURE

In accordance both with sound engineering practice and the quality assurance commonly applied in the nuclear industry, the development of computer programs and procedures for analysis must be accompanied by adequate documentation and verification of the accuracy of results. The procedures used for verification of results from computer programs NUPITE and NUPITE II were discussed in the previous sections. Adequate documentation for each program has been carried out by the compilation of three documents: (a) a program users manual, describing procedures for performing the analysis and input requirements; (b) a programmers manual, which progresses from general organization of the programs into overlays and subroutines through definition of variables and detailed flow charting of each subroutine; and (c) the program verification manual. A design control procedure is used to regulate changes in either the documentation or the programs themselves.

A number of opportunities have transpired for comparison of calculated results with those measured for piping in the operating condition. In general, it has been found that calculated and measured thermal deflections agreed closely. However, the accuracy of prediction of piping response to dynamic loads, such as steam hammer, water hammer, and for vibratory motion has been found highly sensitive to the accuracy in modeling the piping support conditions. The effects of stiffness variations in nonlinear piping restraints inherent in some types of piping supports may be substantial.

REFERENCES

9 Naval Ship Research and Development Center, "MEL-10, Pipe Flexibility Analysis", Washington, D. C., 1957.


